

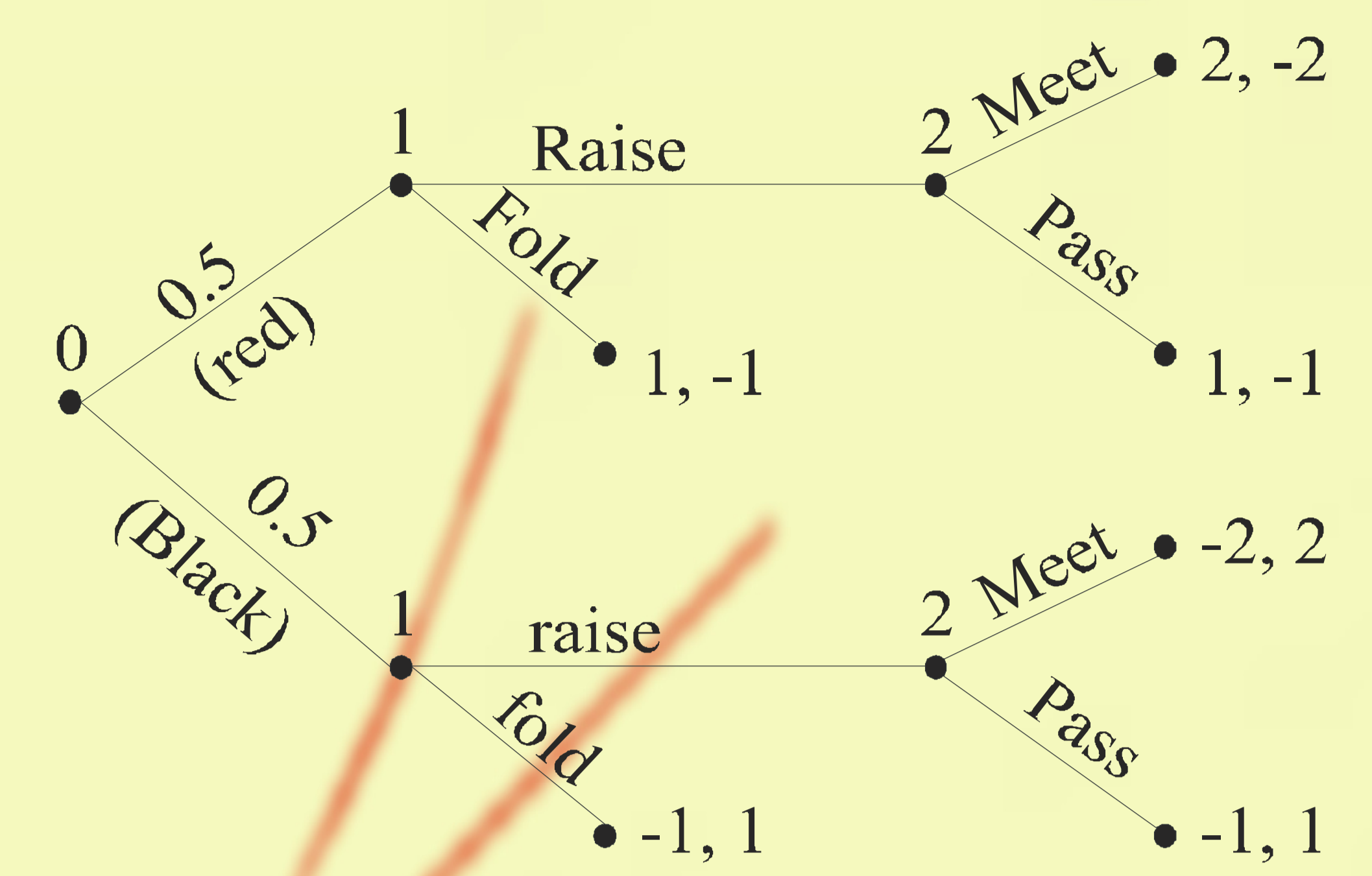
# GAME THEORY AND INCENTIVES

## history of game theory

folk wisdom *the Holy Bible, Talmud*  
 combinatorial games *Pascal, Bernoulli* (16th century)



1913 Ernst Zermelo  
*chess as a zero sum game*  
 1921 Emile Borel *minmax games*  
 1928 John von Neumann *minmax theorem*  
 1942 the Michael Curtiz film *Casablanca*  
*an example of real life games*  
 1944 John von Neumann & Oscar Morgenstern  
*Theory of Games and Economic Behavior*  
 1950 John Nash introduces Nash equilibrium concept  
 1953 Lloyd Shapley introduces Shapley value for cooperative games



## Nash equilibrium

1953 *prisoner's dilemma game*  
 Harold W. Kuhn & Alan W. Tucker

	not confess	confess
not confess	1, 1	6, 0
confess	0, 6	5, 5

number of years in prison



players 1 and 2,  
 actions  $x, y$  and  
 profits  $\pi_1(x, y), \pi_2(x, y)$

reaction curves  $R_1(y), R_2(x)$

$$\pi_1(R_1(y), y) = \max_x \pi_1(x, y)$$

$$\pi_2(x, R_2(x)) = \max_y \pi_2(x, y)$$

Nash equilibrium  $x^N, y^N$

$$x^N = R_1(y^N), y^N = R_2(x^N)$$

## Nobel laureates in

John Nash  
*Nash equilibrium*  
 John Harsanyi  
*incomplete information, Bayesian games, 1967*  
 Reinhard Selten  
*dynamic games*  
*subgame-perfect equilibrium, 1965*  
*prisoner's dilemma revisited*



1994

2000 Game Theory Society is founded  
 2002 the film *Beautiful Mind* about John Nash's life

## computation of $x^N$ and $y^N$

an adjustment

process

to reach the  
 equilibrium

$$\begin{cases} x_{k+1} = R_1(y_k) \\ y_{k+1} = R_2(x_{k+1}) \end{cases}$$

## the International Society of Dynamic Games

ISDG

founded in Otaniemi 1990

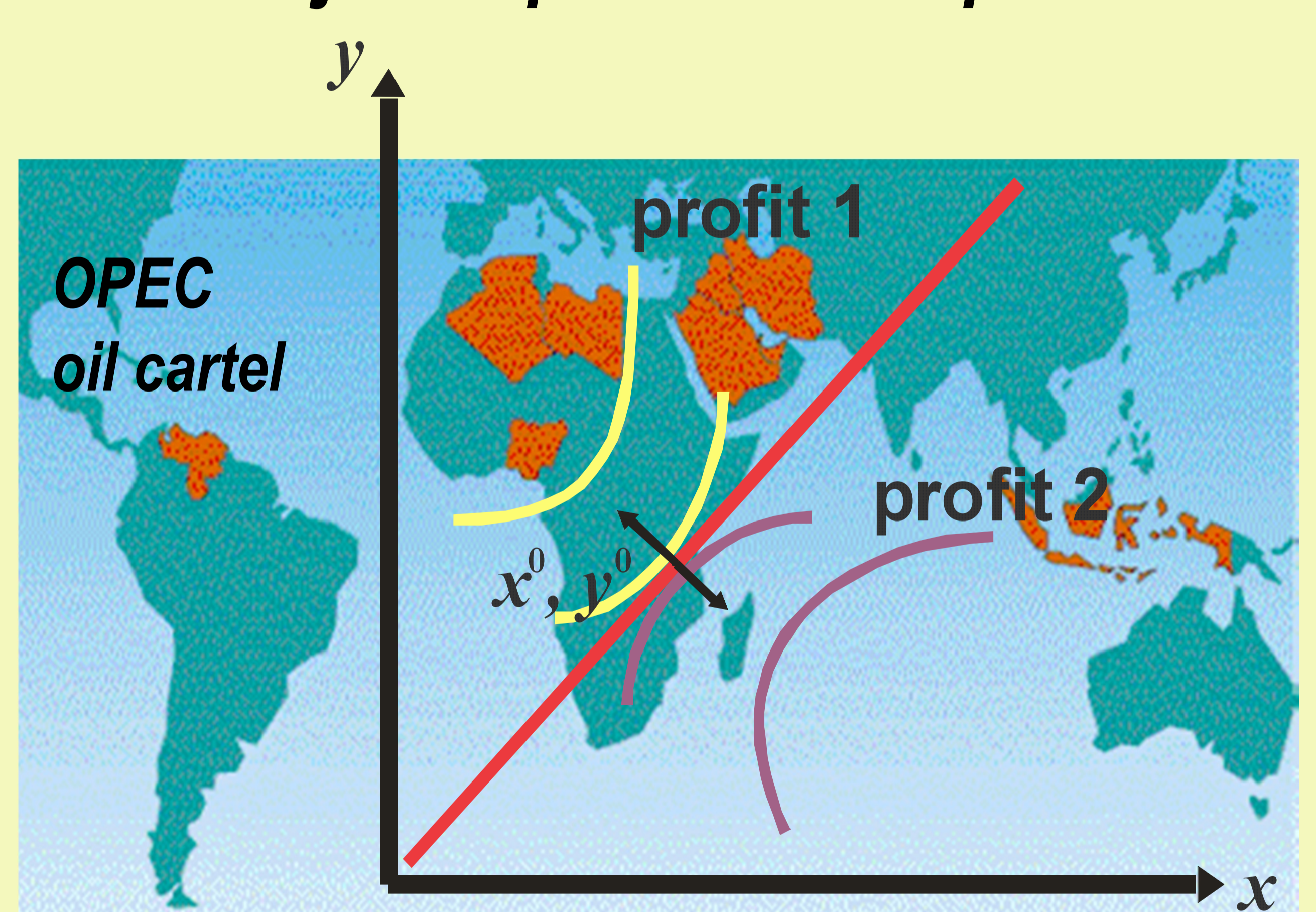
## incomplete information and Bayesian games

players with unforeseeable behaviour enter the scene

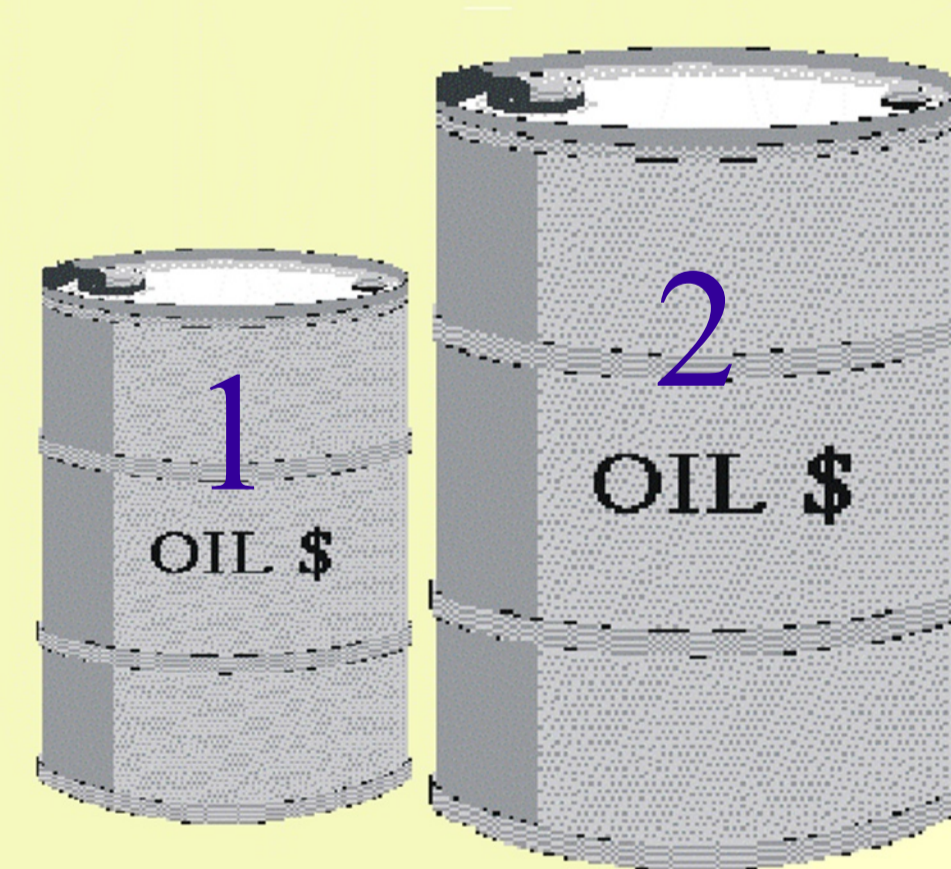


nobody knows  
 the other players'  
 true intentions,  
 their types...  
 yet, they must  
 play the game

## Osborne's quota rule makes the joint optimum an equilibrium



## cartel example



two countries, joint optimum  $x^0, y^0$   
 line of constant market shares  
 $x/y = x^0/y^0$   
 maintaining their market shares  
 keeps the countries at  $x^0, y^0$

Osborne's rule is an example of an incentive equilibrium  
 in our research the rule is generalized to dynamic games

in SAL we study tariff design in buyer-seller games and develop practical schemes to compute the Bayesian-Nash equilibrium

## selected publications

K. Berg and H. Ehtamo: Continuous learning methods in two-buyer pricing problem, *Mathematical Methods of Operations Research*, 2012  
 K. Berg and H. Ehtamo: Interpretation of Lagrange multipliers in nonlinear pricing problem, *Optimization Letters*, 2010  
 H. Ehtamo, K. Berg and M. Kitti: An adjustment scheme for nonlinear pricing problem with two buyers, *European Journal of Operational Research*, 2010  
 M. Kitti: Convergence of iterative tatonnement without price normalization, *Journal of Economic Dynamics and Control*, 2010  
 M. Kitti and H. Ehtamo: *Osborne's cartel maintaining rule revisited*, Manuscript, 2009  
 M. Kitti and H. Ehtamo: *Adjustment of an Affine Contract with Fixed-Point Iteration*, *Journal of Optimization Theory and Applications*, 2009  
 H. Ehtamo, R.P. Härmäläinen, P. Heiskanen, J. Teich, M. Verkama and S. Zions: *Generating Pareto solutions in two-party negotiations by adjusting artificial constraints*, *Management Science*, 2000  
 M. Verkama, H. Ehtamo and R.P. Härmäläinen: *Distributed computation of Pareto solutions in N-player games*, *Mathematical Programming*, 1996  
 H. Ehtamo and R.P. Härmäläinen: *A cooperative incentive equilibrium for a resource management problem*, *Journal of Economic Dynamics and Control*, 1993  
 H. Ehtamo and R.P. Härmäläinen: *Incentive strategies and equilibria for dynamic games with delayed information*, *Journal of Optimization Theory and Applications*, 1989